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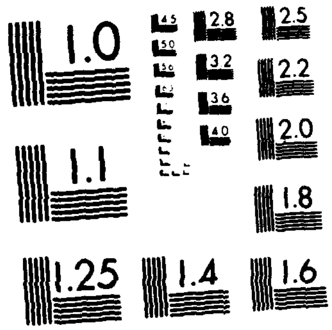

DEVELOPMENT OF A MORE GENERAL REYNOLDS STRESS CLOSURE FOR SWIRLING FLOW

FINAL REPORT

March 1988

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Second order closure, Reynolds stress model, homogeneous turbulence, non-linear effects, swirling flows, compressible flows.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper is concerned with the modelling of the return to isotropy part of the pressure strain term in homogeneous anisotropic turbulent flows. Analytical solutions of the transport equations of the invariants of the anisotropy tensor as well as that of turbulent kinetic energy as a function of the natural time of decay are provided and discussed. Principal components of the Reynolds stresses are obtained from the solution of a cubic equation which involves the invariants. It is shown that current models based on Rotta's

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20 hypothesis are subject to a constraint which is only satisfied by axisymmetric homogeneous turbulence, and the constraint can be eliminated by non-linear modelling. A physical picture of energy transfer among the Reynolds stress components which takes into account the influence of the third invariant on the process of return to isotropy is presented.

The work is clearly unfinished due to administrative difficulty. Further work in the modelling of rotating turbulence, application of Reynolds stress model to compressible flows and swirling flows of engingering relevance were not carried out.

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b. **STATEMENT OF WORK**

The objective of this research is to develop a more general and universally applicable model of the pressure-strain correlation of the Reynolds stress transport equations with special emphasis on turbulent swirling flows. A systematic investigation of homogeneous turbulent flows will be continued with the aim of establishing the functional dependence of the model constants on the second and third invariants of the anisotropy tensor. The model developed will first be applied to the case of homogeneous rotating turbulence and then to more complicated compressible flows and swirling flows of practical importance. An experimental program also is proposed to generate detailed measurements of both the mean and turbulence quantities for model validation purposes.

c. **STATUS OF THE RESEARCH EFFORT**

It is generally recognised that accurate modelling of complex practical flows would require the use of second order closure, and that the week point of the closure is the lack of universality of the model for the pressure-strain correlation Ψ_{ij} in the Reynolds stress transport equations. Chou (1) showed that with the aid of a Poisson equation for the fluctuating pressure, p, the pressure-strain terms can be re-expressed in terms of an integral without the explicit appearance of p. The form of the integral suggests that three distinct processes contribute to the pressure-strain terms (1,2). The first, the 'return to isotropy' term, $\Psi_{ij,1}$, involves only the fluctuating velocities. The second, the 'rapid term' or the 'isotropization of the production term' $\Psi_{ij,2}$, involves the interaction between the mean strain and fluctuating velocities. The third, the wall proximity term, $\Psi_{ij,w}$, expresses the effect of walls on the pressure-strain term. The net effect of these processes is to redistribute the energy among the Reynolds stress components, but nothing can be deduced from the Reynolds stress transport equation about the direction and the speed of the energy transfer.

During the course of this work, substantial achievements have been obtained in the modelling of homogeneous anisotropic turbulence without mean velocity gradient. Experiments (3-6) showed that this kind of flow embodies the general dynamics of the decay of the energy containing eddies, a process described by the pressure strain tensor. The flow also enables the analysis of energy transfer and functional dependence of contants to be simplified as only the return to isotropy part of the pressure-strain tensor remains. Under these conditions, a general form of the current pressure-strain model incorporating non-linear terms (7-9) is as follows:

$$\Psi_{ij} = \left\langle \frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right\rangle$$

$$= -c_1 \epsilon b_{ij} + \lambda \epsilon \left(b_{ik}b_{kj} - \frac{1}{3} II \delta_{ij}\right) \qquad (1)$$

with

$$b_{ij} = \frac{\langle u_i u_j \rangle}{k} - \frac{2}{3}\delta_{ij} \quad ; \quad k = \frac{1}{2}\langle u_i u_j \rangle$$

$$II = b_{ik}b_{ki}$$

where b_{ij} is the anisotropy tensor, k the mean turbulent kinetic energy, II the second invariant of the anisotropy tensor, ϵ the viscous dissipation rate, δ_{ij} the kronecker delta, c_1 and λ may be either constants or functions of some relevant parameters, and angled brackets denote averages over time. The first term on the right hand side of Eq. (1) is the linear model proposed by Rotta (10), and the second term expresses the non-linear effects.

Closure of the Reynolds stress equation requires further modelling assumptions in its viscous dissipation term and in constructing a length scale or the dissipation transport equation. Here, a novel approach has been adopted to isolate and to study the variation of Reynolds stress components along the flow, free from the implications of other modelling assumptions.

It may be shown empirically that the transport equations for turbulent kinetic energy and those for the second and third invariants of the anisotropy tensor contain information on the distribution of energy among the Reynolds stress components. To avoid implications arising from the approximate modelling of the dissipation equation, a non-dimensioinal time τ related to the natural decay time of turbulence energy is introduced.

$$\frac{\epsilon}{k}\, dt = d\tau \;, \text{ with } \tau = -\ln(k/k_0) \text{ and } dt = \frac{dx}{U}$$

where k_0 is the initial value of the turbulent kinetic energy. Then the equations for the turbulent kinetic energy and the invariants of the anisotropy tensor can be expressed as follows:

$$\frac{\partial k}{\partial \tau} = k \qquad => k = k_0 e^{-\tau} \qquad (2)$$

$$\frac{\partial II}{\partial \tau} = -2[(c_1 - 1)II - \lambda\, III] \qquad (3)$$

$$\frac{\partial III}{\partial \tau} = -3[(c_1 - 1)III - \frac{\lambda}{6} II^2] \qquad (4)$$

where $III = b_{ik}b_{kj}b_{ji}$, the third invariant of the anisotropy tensor. By taking the normalised time τ from experiments, the problem is transformed into the solution of three scalar equations with the normalised time as the independent variable. Thus by the solution of Eqs. (2) to (4) the implications of the modelling assumptions of the dissipation equation have been eliminated and the effects of pressure-strain term isolated.

The second invariant, II, can be interpreted as a measure of the state of anisotropy of the flow. The role and physical meaning of the third invariant in the redistribution of energy among the components of the Reynolds stress tensor can be obtained from the solution of the cubic equation for the principal Reynolds stress components, viz.

$$b^3 - 1/2 II b - 1/3 III = 0 \qquad (5)$$

with b representing the individual component of the anisotropy tensor. The solutions show that the sign of the third invariant remains constant during the process of return to isotropy. When the third invariant is negative, the two larger components are giving energy to the smaller, consequently the smallest component tend to increase its energy, while when the third invariant is positive, the two smaller components are gaining energy from the largest and the return to isotroy is slower than the former case.

Analytical solutions of Eqs. (2) to (4) as a function of the normalised time were compared with experimental data. By neglecting non-linear effects (λ=o) and making c_1 a constant, Rotta's (10) proposal is recovered. The solutions is shown to be true only for homogeneous axisymmetric turbulent flows or for flows with small departure from axisymmetry. Similar solution also is obtained when c_1 is not a constant (λ=o). Therefore any improvement in the modelling of the return to isotropy part of the pressure-strain term, in expressing c_1 as a function of some relevant parameters (11), is still unsatisfactory to model non-axisymmetric flows. Optimisation of the value of the constant c_1 against the data of Uberoi (3) (anisotropic decaying turbulence submitted to axisymmetric plan strain with different contraction ratios) showed that the value varies with contraction ratios as well as development of the flow. However Fig. 1 shows that a constant value with c_1 = 2.0 gives satisfactory agreement with the data of Uberoi (3).

When non-linear term in the pressure-strain tensor is considered, the axisymmetric flow experiment of Uberoi (3) can be used to estimate the value of the second constant λ as a function of that of c_1. Figure 2 shows the straight line relationship between quantity $|\lambda / II_0|$ and (c_1-1). Employing different values for c_1 and λ from Fig. 2, the

non-linear model predictions of Uberoi (3) experiment are shown in Fig. 1 where an improvement with respect to the linear model can be noted.

For non-axisymmetric flows, an approximate analytical solution was obtained using perturbation methods. The solutions indicate that λ and c_1-1 are important parameters which influence the speed at which the return to isotropy is taking place. A new parameter which measures the initial degree of departure from axisymmetry also can be defined as:

$$\Omega = 1 - 6 III_0^2 \, II_0^{-3} \qquad (6)$$

so that $\Omega=0$ for axisymmetry and $\Omega \to 1$ as the departure from axisymmetry increases. On an empirical basis, the speed of the process of equilibration of energy can be expressed by the relationship between c_1-1 and the third invariant as

$$c_1 - 1 = e^{-f\Omega III_0} \qquad (7)$$

with f being a constant with a value of 10.

Three return to isotropy experiments were used to compare with the perturbation and the linear model solutions.

Figure 3 shows the comparison for the Tucker and Reynolds (4) experiment where homogeneous grid generated turbulence was subjected to a uniform plane strain. Good agreement is obtained with the perturbation solution in particular for the smallest component which increases its energy along the parallel duct, demonstrating the superiority of the non-linear model.

Gence and Mathieu (5) considered the return to isotropy phenomenon after the homogeneous turbulent flow has been subjected to two successive plane strains whose principal axes are shifted by an angle α. Figure 4 shows that when $\alpha=o$ and $\alpha=\pi/4$, excellent agreement is obtained with perturbation solution. When $\alpha=3\pi/4$, the perturbation solution becomes less satisfactory but this may be attributed to the uncertainty in applying the values of c_1 and λ, which have been found from initial conditions, to the overall state of the flow.

Le Penven et al (6) observed the evolution of turbulent kinetic energy, second invariant and the Reynolds stress components, under the influence of the third invariant with different signs. Figure 5 shows that when the third invariant is negative, better agreement is obtained with perturbation solution then with the linear model, especially for the smallest component. When the third invariant is positive both solutions seem to be unsatisfactory, but again there appears to be some

uncertainty in the initial time of decay.

In conclusion, a breakthrough has been achieved in the understanding of the dynamics of energy transfer and the non-linear effects in the return to isotropy process of the pressure-strain term. The use of non-linear term is expected to eliminate the constraint of axisymmetry imposed by linear modelling of Rotta (10).

The work is clearly unfinished due to administrative difficulty. It was proposed to model homogeneous turbulence with mean strain, especially that of a rotating turbulence in order to study the functional dependence of the constants and also the non-linear effects in the 'rapid term' or isotropization of production process in the pressure strain tensor. Then the Reynolds stress model would be employed to model compressible flows and swirling flows of engineering relevance.

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d. **PUBLICATIONS**

1. Vasquez-Malebran, S.A. and Boysan, F. (1985) 'On the modelling of the return to isotropy of homogeneous non-isotropic turbulence', Fifth Symposium on Turbulent Shear Flows, Cornell University, p. 12.25-12.30.

2. Swithenbank, J. (1987) 'Mixing problems in supersonic combustion', US-France Workshop on Turbulent Reacting Flows, Rouen, France.

e. **PROFESSIONAL PERSONNEL ASSOCIATED WITH THE RESEARCH EFFORT**

Boysan, F.
Chin, S.B.
Swithenbank, J.
Vasquez-Malebran, S.A. -Ph.D. thesis in preparation

f. **INTERACTION**

1. Presented 'On the modelling of the return to isotropy of homogeneous non-isotropic turbulence', 5th Symposium on Turbulent Shear Flows, Cornell University, 1985.

2. 'Mixing problems in supersonic combustion', presented at US-France Workshop on Turbulent Reacting Flows, Rouen, France, July 1987.

g. **NEW DISCOVERIES**

The work carried out can be regarded as an important breakthrough in Reynolds stress modelling. We are now able to model highly anisotropic turbulent flows with a clearer understanding of the non-linear effects and functional dependence of the values of the constants.

h. **ANY OTHER STATEMENTS**

See section g.

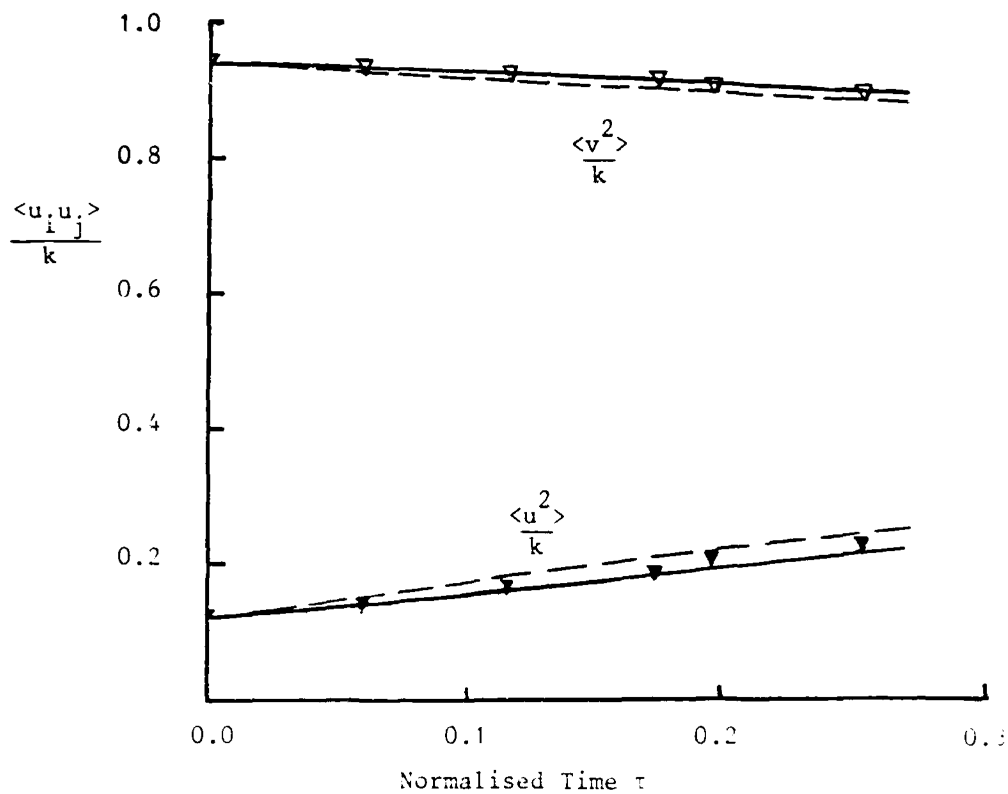

Figure 1 Comparison of Reynolds stresses in Uberoi experiments with linear (---) and non-linear (——) solutions.

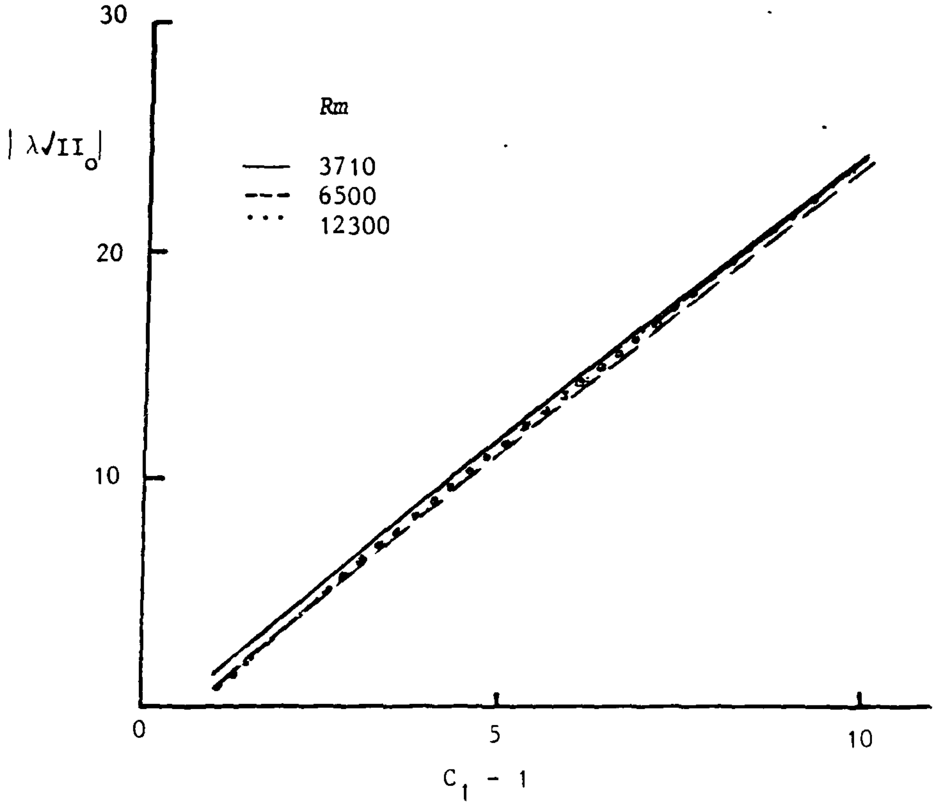

Figure 2 Relationship between λ and $c_1 - 1$
Uberoi experiment. Contraction ratio 4:1

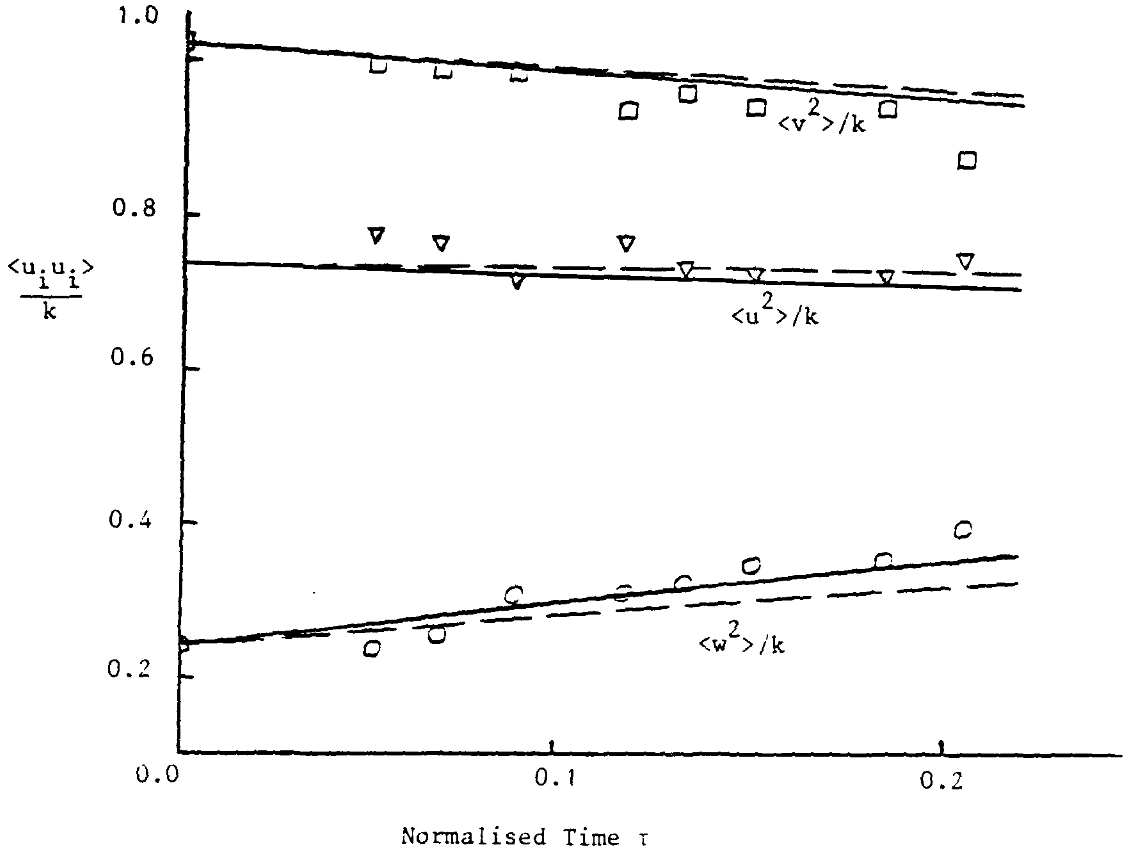

Figure 3 Comparison of Reynolds stresses in the Tucker and Reynolds experiments with linear (---) and non-linear (——) solutions.

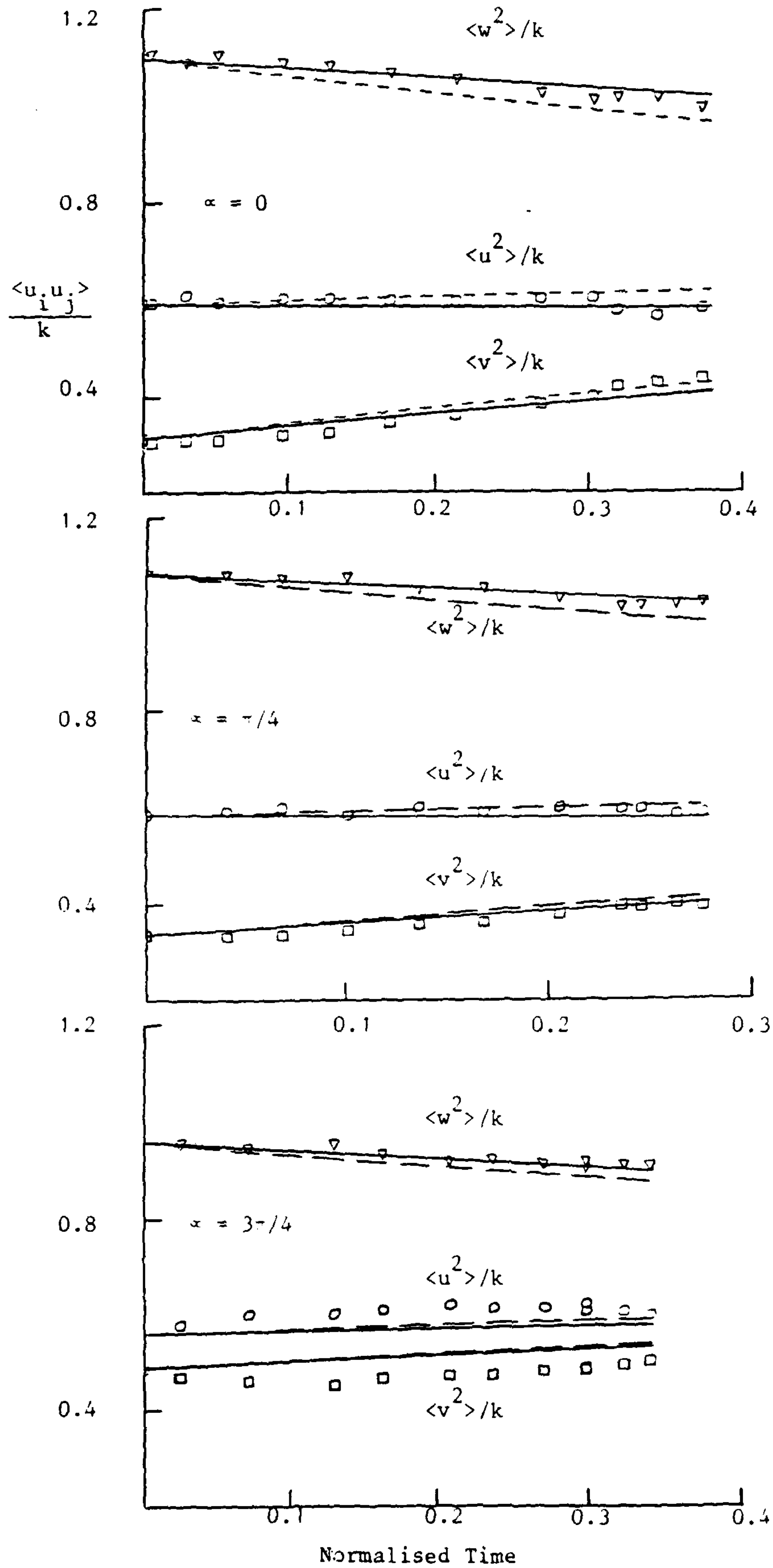

Figure 4 Comparison of Reynolds stresses in Gence and Mathieu experiments with linear (---) and non-linear (——) solutions.

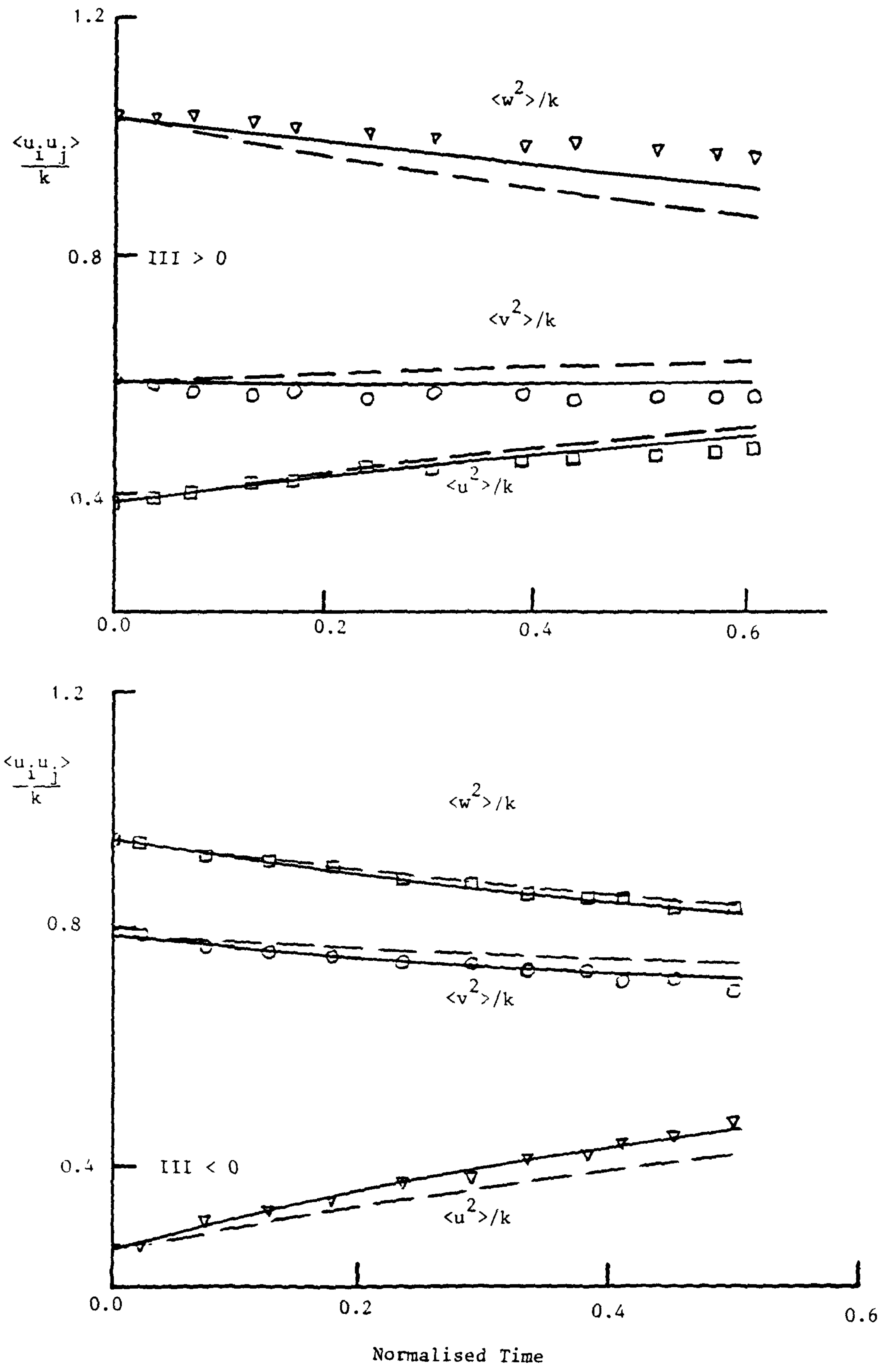

Figure 5 Comparison of Reynolds stresses in Le Penven et al experiment with linear (---) and non-linear (——) solutions.

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